Zadatak 1 *S točnošću većom od 10^{-3} odredite $\cos 213°$. Izračunajte ukupnu grešku.*

Rješenje.

$$\cos 213° = \cos(180° + 33°) = -\cos 33° = -\cos\frac{11\pi}{60} \Rightarrow x = \frac{11\pi}{60}$$

$$n = 2 \Rightarrow R_6\left(\frac{11\pi}{60}\right) \le \frac{\left(\frac{11\pi}{60}\right)^6}{6!} = 0.507 \cdot 10^{-4} < 0.25 \cdot 10^{-4}$$

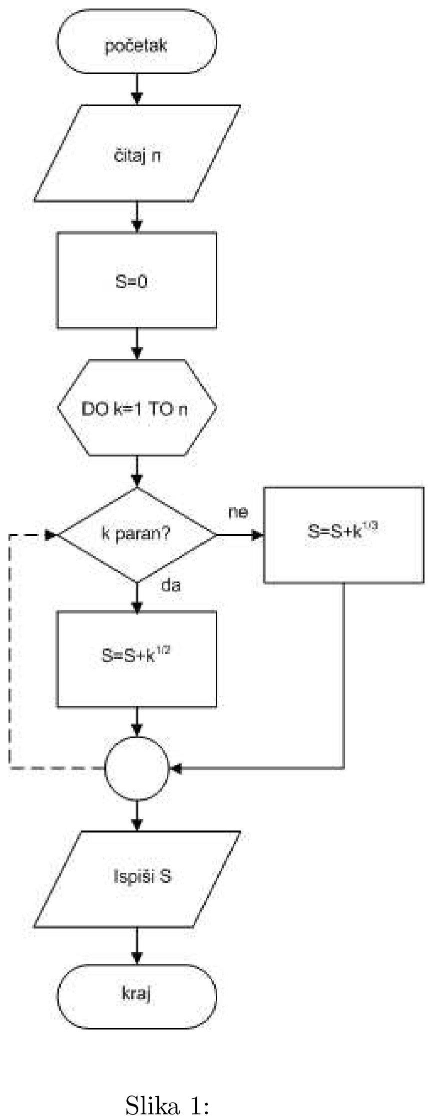
$$\Rightarrow -\cos\frac{11\pi}{60} = -1 + \frac{1}{2}\left(\frac{11\pi}{60}\right)^2 - \frac{1}{24}\left(\frac{11\pi}{60}\right)^4 = -1 + 0.1659 - 0.0046 = -0.8387$$

$$\varepsilon = 0.507 \cdot 10^{-4} + 2 \cdot 0.5 \cdot 10^{-4} + 0 = 0.1507 \cdot 10^{-3} < 10^{-3}.$$

Zadatak 2 *Opišite dijagram toka i napišite program u Mathematica-i za algoritam koji za zadani cijeli broj $n \ge 1$ (ulazna informacija) računa*

$$\begin{array}{ll} 1 + \sqrt{2} + \sqrt[3]{3} + \sqrt{4} + \sqrt[3]{5} + \sqrt{6} + \cdots + \sqrt[3]{n} & n \text{ neparan}, \\ 1 + \sqrt{2} + \sqrt[3]{3} + \sqrt{4} + \sqrt[3]{5} + \sqrt{6} + \cdots + \sqrt{n} & n \text{ paran}. \end{array}$$

Rješenje.



Slika 1:

```
n = 100;
S = 0;
For[k = 1, k ≤ n, k = k + 1,
 If[IntegerQ[k/2] == True, S = S + N[√k], S = S + N[∛k]]]
Print[S]
```

512.178

Zadatak 3 *Gauss-Seidelovom metodom (jednom iteracijom) odredite približno rješenje sustava*

$$\begin{aligned} 6x_1 + 2x_2 &= 4 \\ x_1 + 2x_2 &= 3. \end{aligned}$$

Odredite pravu grešku.

Rješenje.

$$L = \begin{bmatrix} 6 & 0 \\ 1 & 2 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\Rightarrow x^{(0)} = L^{-1}b = \frac{1}{12}\begin{bmatrix} 2 & 0 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{7}{6} \end{bmatrix}$$

$$\Rightarrow x^{(1)} = \frac{1}{12}\begin{bmatrix} 2 & 0 \\ -1 & 6 \end{bmatrix} \left(\begin{bmatrix} 4 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{3} \\ \frac{7}{6} \end{bmatrix} \right) = \begin{bmatrix} \frac{5}{18} \\ \frac{49}{36} \end{bmatrix}$$

Pravo rješenje:

$$\begin{aligned} 6x_1 + 2x_2 &= 4 \\ x_1 + 2x_2 &= 3 \end{aligned} \Leftrightarrow \begin{aligned} 6x_1 + 2x_2 &= 4 \\ -6x_1 - 12x_2 &= -18 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ \frac{7}{5} \end{bmatrix}$$

Prava greška:

$$\varepsilon = \sqrt{\left(\frac{5}{18} - \frac{1}{5}\right)^2 + \left(\frac{49}{36} - \frac{7}{5}\right)^2} = 0.087$$

Zadatak 4 *Odredite vezu oblika* $y = e^{\frac{a}{x-b}}$ *ako je*

x_k	0	1	2
y_k	0.5	0.4	0.1

Rješenje.

$$\ln y = \frac{a}{x-b} \Rightarrow \frac{1}{\ln y} = \frac{x-b}{a} \Rightarrow \bar{y} = a_0 + a_1\bar{x}, \quad \bar{y} = \frac{1}{\ln y}, \bar{x} = x, a_0 = -\frac{b}{a}, a_1 = \frac{1}{a}$$

$\bar{x}_i$	0	1	2
$\bar{y}_i$	−1.44	−1.09	−0.43

$$\Rightarrow \sum_{i=0}^{2} \bar{x}_i = 3, \sum_{i=0}^{2} \bar{x}_i^2 = 5, \sum_{i=0}^{2} \bar{x}_i\bar{y}_i = -1.95, \sum_{i=0}^{2} \bar{y}_i = -2.96 \Rightarrow a_0 = -1.49, a_1 = 0.505$$

$$\Rightarrow a = \frac{1}{a_1} = 1.98, \ b = -a_0 a = 2.95 \Rightarrow y = e^{\frac{1.98}{x-2.95}}$$

Zadatak 5 *Odredite trigonometrijski polinom prvog stupnja koji u smislu metode najmanjih kvadrata najbolje aproksimira funkciju* $f(x) = |\frac{x}{3}|, \ x \in [-\pi, \pi]$. *Odredite kvadratnu grešku te aproksimacije.*

Rješenje.

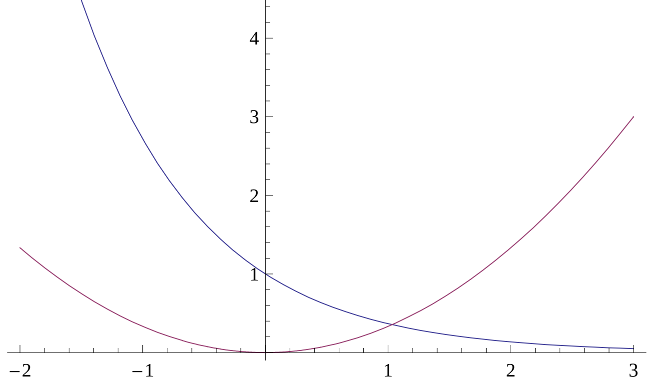
$$L = \pi, \quad A_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -\frac{x}{3}dx + \frac{1}{2\pi}\int_{0}^{\pi} \frac{x}{3}dx = \frac{\pi}{6}$$

$$A_1 = \frac{1}{\pi}\int_{-\pi}^{0} -\frac{x}{3}\cos x dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{x}{3}\cos x dx = -\frac{4}{3\pi}, \ B_1 = \frac{1}{\pi}\int_{-\pi}^{0} -\frac{x}{3}\sin x dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{x}{3}\sin x dx = 0$$

$$T_1(x) = \frac{\pi}{6} - \frac{4}{3\pi}\cos x, \quad \varepsilon_1 = \int_{-\pi}^{\pi}\frac{x^2}{9}dx - 2\pi\left[\frac{\pi^2}{36} + \frac{1}{2}\frac{16}{9\pi^2}\right] = 0.00836$$

Zadatak 6 *Za jednadžbu* $e^{-x} = \frac{x^2}{3}$ *odredite funkciju* φ *s kojom se može provesti metoda iteracije.*

Rješenje.



Slika 3:

$$f(x)=e^{-x}-\frac{x^2}{3},\quad f(1)=0.03>0,\quad f(2)=-1.2<0 \Rightarrow \text{ nultočka je unutar intervala } [1,2]$$

$$x=\sqrt{3e^{-x}} \Rightarrow \varphi(x)=\sqrt{3e^{-x}} \Rightarrow \varphi'(x)=-\frac{1}{2\sqrt{3}}e^{-\frac{x}{2}}$$

$$\Rightarrow |\varphi'(x)|<1.$$

Zadatak 7 *Newtonovom metodom (jednom iteracijom) odredite približno rješenje sustava*

$$\begin{array}{rcl} xy-y-1 & = & 0 \\ x^2-y^2-1 & = & 0 \end{array},$$

uzimajući za početne vrijednosti $x_0=2,\ y_0=1$.

Rješenje.

$$F(X)=\begin{bmatrix} xy-y-1 \\ x^2-y^2-1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Rightarrow F'(X)=\begin{bmatrix} y & x-1 \\ 2x & -2y \end{bmatrix} \Rightarrow [F'(x)]^{-1}=\frac{1}{\det[F'(X)]}\begin{bmatrix} -2y & 1-x \\ -2x & y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}=\begin{bmatrix} 2 \\ 1 \end{bmatrix}-\frac{1}{-6}\begin{bmatrix} -2 & -1 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}=\begin{bmatrix} 1.66667 \\ 1.33333 \end{bmatrix}$$