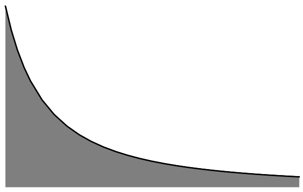
MATEMATIKA II - 3.test GRUPA A1 18.04.2006.

1. Izračunajte površinu područja određenog sa $0 \leq y \leq \frac{1}{x(x-3)}$, $4 \leq x \leq 10^5$.
   Rješenje:

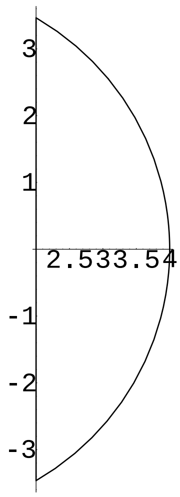


$$P = \int_4^{10^5} \left[\frac{1}{x(x-3)} - 0 \right] dx = \int_4^{10^5} \frac{1}{3} \left[\frac{1}{x-3} - \frac{1}{x} \right] dx = \frac{1}{3} \ln \frac{x-3}{x} \bigg|_4^{10^5}$$

$$= \frac{1}{3} \left[\ln \frac{10^5 - 1}{10^5} - \ln \frac{1}{4} \right] \approx 0.462088.$$

□

2. Izračunajte opseg lika zadanog u polarnim koordinatama sa $\frac{2}{\cos\varphi} \leq r \leq 4$.
   Rješenje: Iz $\frac{2}{\cos\varphi} = 4 \Leftrightarrow \cos\varphi = \frac{1}{2} \Rightarrow \varphi = \pm\frac{\pi}{3}$ i formule za duljinu luka



krivulje zadane u polarnim koordinatama slijedi

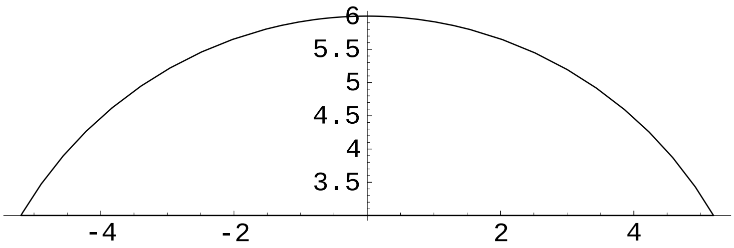
$$O = s_1 + s_2 = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{4^2 + 0^2}\, d\varphi + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{\frac{4}{\cos^2\varphi} + \left(\frac{2\sin\varphi}{\cos^2\varphi} \right)^2}\, d\varphi$$

$$= \frac{8\pi}{3} + \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{d\varphi}{\cos^2\varphi} = \frac{8\pi}{3} + 2\,\mathrm{tg}\,\varphi \bigg|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \frac{8\pi}{3} + 4\sqrt{3} \approx 15.3058.$$

□

MATEMATIKA II - 3.test GRUPA B1 18.04.2006.

1. Izračunajte opseg lika zadanog u polarnim koordinatama sa $\frac{3}{\sin\varphi} \le r \le 6$.
   Rješenje: Iz $\frac{3}{\sin\varphi} = 6 \Leftrightarrow \sin\varphi = \frac{1}{2} \Rightarrow \varphi = \frac{\pi}{6}, \frac{5\pi}{6}$ i formule za duljinu luka



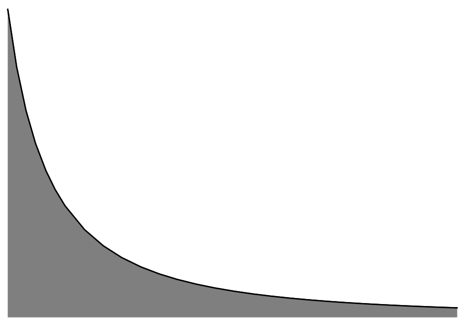
   krivulje zadane u polarnim koordinatama slijedi

$$O = s_1 + s_2 = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{6^2 + 0^2}\, d\varphi + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sqrt{\frac{9}{\sin^2\varphi} + \left(-\frac{3\cos\varphi}{\sin^2\varphi}\right)^2}\, d\varphi$$

$$= 4\pi + \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{d\varphi}{\sin^2\varphi} = 4\pi - 3\operatorname{ctg}\varphi \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = 4\pi + 6\sqrt{3} \approx 22.9587.$$

□

2. Izračunajte površinu područja određenog sa $0 \le y \le \frac{1}{(x+3)x}$, $1 \le x \le 10^4$.
   Rješenje:



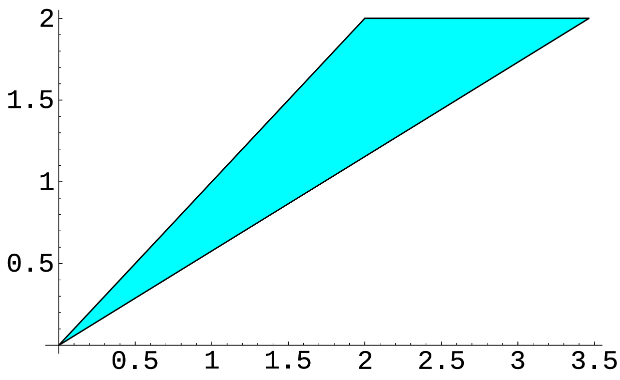
$$P = \int_1^{10^4} \left[\frac{1}{x(x+3)} - 0\right] dx = \int_1^{10^4} \frac{1}{3}\left[\frac{1}{x} - \frac{1}{x+3}\right] dx = \frac{1}{3} \ln \frac{x}{x+3}\Big|_1^{10^4}$$

$$= \frac{1}{3}\left[\ln\frac{10^4}{10^4+3} - \ln\frac{1}{4}\right] \approx 0.461998.$$

□

MATEMATIKA II - 3.test GRUPA A2 19.04.2006.

1. Koristeći polarne koordinate izračunajte površinu trokuta određenog točkama $A(0,0)$, $B(2,2)$, $C(2\sqrt{3},2)$.
   Rješenje: Kako je jednadžba pravca $y=2$ u polarnim koordinatama $r=\frac{2}{\sin\varphi}$,

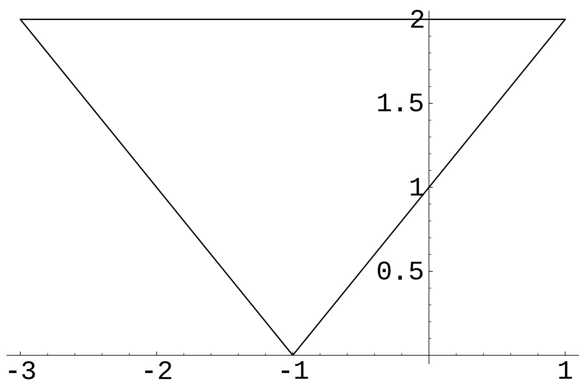


te kako pravci kroz ishodište i točke $B(2,2)$ te $C(2\sqrt{3},2)$ imaju koeficijente smjera $k_1=1=\operatorname{tg}\frac{\pi}{4}$, $k_2=\frac{\sqrt{3}}{3}=\operatorname{tg}\frac{\pi}{6}$, to slijedi

$$P=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\frac{4}{\sin^2\varphi}d\varphi=2\left[-\operatorname{ctg}\varphi\right]\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}=2\left[\operatorname{ctg}\frac{\pi}{6}-\operatorname{ctg}\frac{\pi}{4}\right]=2(\sqrt{3}-1)\approx 1.4641.$$

□

2. Koristeći integralni račun izračunajte opseg lika određenog sa $|x+1|\leq y\leq 2$.
   Rješenje: Kako rubne krivulje imaju jednadžbe $y=x+1$ za $x\in[-1,1]$,



$y=-(x+1)$ za $x\in[-3,-1]$, $y=2$ za $x\in[-3,1]$ to slijedi

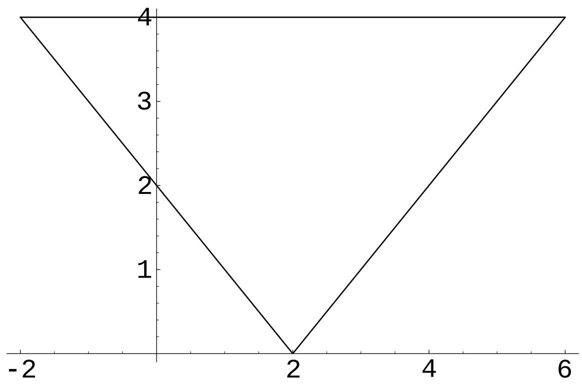
$$O=s_1+s_2+s_3=\int_{-1}^{1}\sqrt{1+1^2}\,dx+\int_{-3}^{-1}\sqrt{1+(-1)^2}\,dx+\int_{-3}^{1}\sqrt{1+0^2}\,dx$$

$$=4\sqrt{2}+4=4(1+\sqrt{2})\approx 9.65685.$$

□

MATEMATIKA II - 3.test GRUPA B2 19.04.2006.

1. Koristeći integralni račun izračunajte opseg lika određenog sa $|x-2| \le y \le 4$.
Rješenje: Kako rubne krivulje imaju jednadžbe $y = x - 2$ za $x \in [2, 6]$,



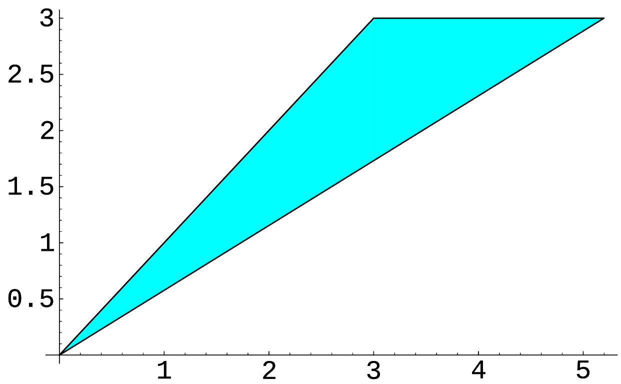
$y = -(x-2)$ za $x \in [-2, 2]$, $y = 6$ za $x \in [-2, 6]$ to slijedi

$$O = s_1 + s_2 + s_3 = \int_2^6 \sqrt{1 + 1^2}\, dx + \int_{-2}^2 \sqrt{1 + (-1)^2}\, dx + \int_{-2}^6 \sqrt{1 + 0^2}\, dx$$

$$= 8\sqrt{2} + 8 = 8(1 + \sqrt{2}) \approx 19.3137.$$

□

2. Koristeći polarne koordinate izračunajte površinu trokuta određenog točkama $A(0,0)$, $B(3,3)$, $C(3\sqrt{3}, 3)$.
Rješenje: Kako je jednadžba pravca $y = 3$ u polarnim koordinatama $r = \frac{3}{\sin\varphi}$,



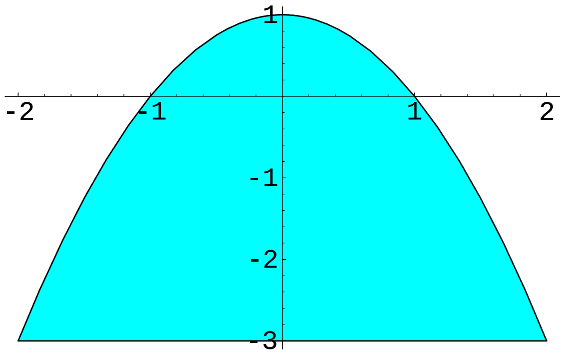
te kako pravci kroz ishodište i točke $B(3,3)$ te $C(3\sqrt{3}, 3)$ imaju koeficijente smjera $k_1 = 1 = \operatorname{tg}\frac{\pi}{4}$, $k_2 = \frac{\sqrt{3}}{3} = \operatorname{tg}\frac{\pi}{6}$, to slijedi

$$P = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{9}{\sin^2\varphi} d\varphi = \frac{9}{2}\left[-\operatorname{ctg}\varphi\right]\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{9}{2}\left[\operatorname{ctg}\frac{\pi}{6} - \operatorname{ctg}\frac{\pi}{4}\right] = \frac{9}{2}(\sqrt{3} - 1) \approx 3.29423.$$

□

MATEMATIKA II - 3.test GRUPA A3 20.04.2006.

1. Izračunajte volumen tijela nastalog rotacijom područja $-3 \leq y \leq 1 - x^2$ oko osi apscisa.
   Rješenje:

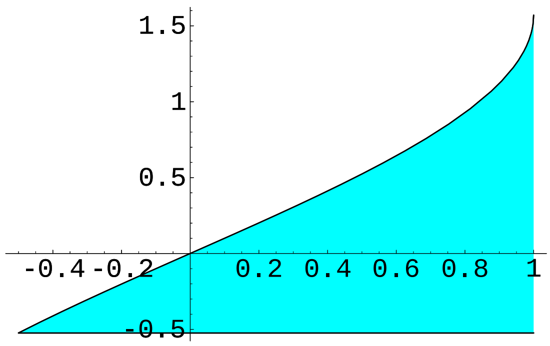


$$V_{y=0} = 2V_1 = 2\pi \left[\int_0^1 (-3)^2 \, dx - \int_1^2 (1-x^2)^2 \, dx \right] = 2\pi \left[9 - \int_1^2 (1 - 2x^2 + x^4) \, dx \right]$$

$$= 2\pi \left[9 - (x - \frac{2}{3}x^3 + \frac{1}{5}x^5) \Big|_1^2 \right] = \frac{194}{15}\pi \approx 40.6313.$$

□

2. Izračunajte površinu područja u ravnini određenog sa $-\frac{\pi}{6} \leq y \leq \arcsin x$.
   Rješenje:



$$P = \int_{-\frac{1}{2}}^1 \left(\arcsin x - (-\frac{\pi}{6}) \right) dx = \int_{-\frac{1}{2}}^1 \arcsin x \, dx + \frac{\pi}{4}$$

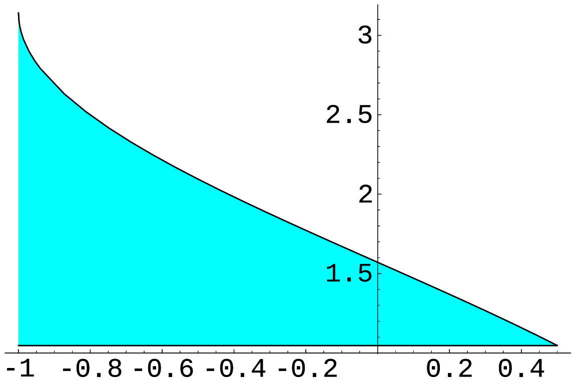
$$= \left\{ \begin{array}{c} u = \arcsin x \Rightarrow du = \frac{dx}{\sqrt{1-x^2}} \\ dv = dx \Rightarrow v = x \end{array} \right\} = x \arcsin x \Big|_{-\frac{1}{2}}^1 - \int_{-\frac{1}{2}}^1 \frac{x dx}{\sqrt{1-x^2}} + \frac{\pi}{4}$$

$$= \arcsin 1 - \left(-\frac{1}{2}\right) \arcsin \left(-\frac{1}{2}\right) + \sqrt{1-x^2} \Big|_{-\frac{1}{2}}^1 + \frac{\pi}{4} = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \approx 1.22837.$$

□

MATEMATIKA II - 3.test GRUPA B3 20.04.2006.

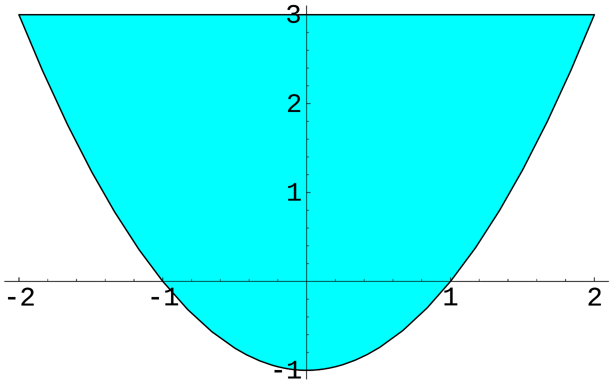
1. Izračunajte površinu područja u ravnini određenog sa $\frac{\pi}{3} \leq y \leq \arccos x$.
Rješenje:



$$P = \int_{-1}^{\frac{1}{2}} \left[\arccos x - \frac{\pi}{3}\right] dx = \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \approx 1.22837.$$

□

2. Izračunajte volumen tijela nastalog rotacijom područja $x^2 - 1 \leq y \leq 3$ oko osi apscisa.
Rješenje:

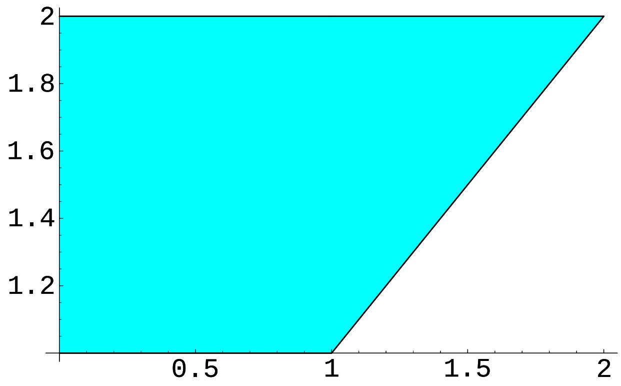


$$V_{y=0} = 2V_1 = 2\pi \left[\int_0^2 3^2 dx - \int_1^2 (x^2 - 1)^2 \, dx\right] = \frac{464}{15}\pi \approx 97.13067.$$

□

MATEMATIKA II - 3.test GRUPA A4 21.04.2006.

1. Koristeći polarne koordinate izračunajte površinu trapeza određenog točkama $A(1,1)$, $B(2,2)$, $C(0,2)$, $D(0,1)$.
   Rješenje: Kako su jednadžbe pravaca $y = 1$, $y = 2$ u polarnim koordinatama

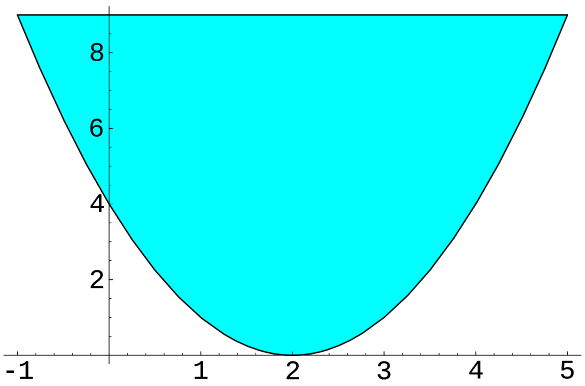


dane sa $r = \frac{1}{\sin\varphi}$, $r = \frac{2}{\sin\varphi}$, a pravci kroz točke A i B te C i D zatvaraju sa pozitivnim smjerom osi apscisa kutove $\frac{\pi}{4}$ i $\frac{\pi}{2}$, to slijedi

$$P = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{4}{\sin^2\varphi} - \frac{1}{\sin^2\varphi}\right) d\varphi = \frac{3}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{d\varphi}{\sin^2\varphi} = \frac{3}{2} = 1.5.$$

□

2. Izračunajte volumen tijela nastalog rotacijom područja $(x-2)^2 \leq y \leq 9$ oko osi ordinata.
   Rješenje:



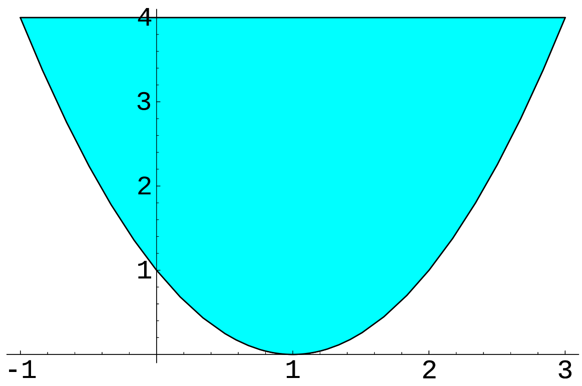
$$V_{x=0} = 2\pi\int_0^5 x\left[9-(x-2)^2\right]\, dx = 2\pi\int_0^5 (5x - x^3 + 4x^2)\, dx$$

$$= 2\pi\left(\frac{5}{2}x^2 - \frac{1}{4}x^4 + \frac{4}{3}x^3\right)\Big|_0^5 = \frac{875}{6}\pi \approx 458.149.$$

□

MATEMATIKA II - 3.test GRUPA B4 21.04.2006.

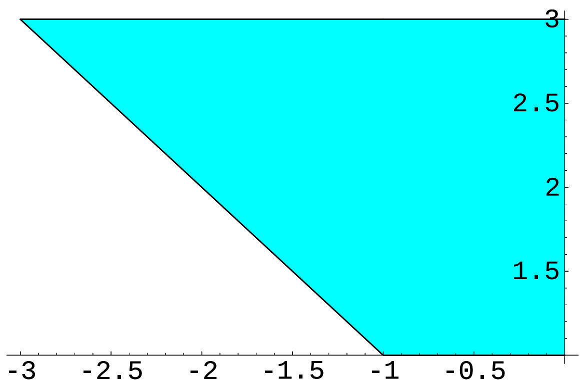
1. Izračunajte volumen tijela nastalog rotacijom područja $(x-1)^2 \leq y \leq 4$ oko osi ordinata.
   Rješenje:



$$V_{x=0} = 2\pi \int_0^3 x\left[4-(x-1)^2\right]\, dx = \frac{45}{2}\pi \approx 70.6858.$$

□

2. Koristeći polarne koordinate izračunajte površinu trapeza određenog točkama $A(0,1)$, $B(0,3)$, $C(-3,3)$, $D(-1,1)$.
   Rješenje: Kako su jednadžbe pravaca $y=1$, $y=3$ u polarnim koordinatama



dane sa $r = \frac{1}{\sin\varphi}$, $r = \frac{3}{\sin\varphi}$, a pravci kroz točke A i B te C i D zatvaraju sa pozitivnim smjerom osi apscisa kutove $\frac{\pi}{2}$ i $\frac{3\pi}{4}$, to slijedi

$$P = \frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{9}{\sin^2\varphi} - \frac{1}{\sin^2\varphi}\right) d\varphi = 4\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \frac{d\varphi}{\sin^2\varphi} = 2.$$

□