Zadatak 1 *S točnošću većom od* 10^{-3} *odredite* $\ln 51$. *Izračunajte ukupnu grešku.*

Rješenje.

$$51 = 2^6 \cdot \frac{51}{64} \Rightarrow \frac{1-x}{1+x} = \frac{51}{64} \Rightarrow x = \frac{13}{115}$$

$$n = 1 \Rightarrow R_3\left(\frac{13}{115}\right) \leq \frac{9}{4} \cdot \frac{\left(\frac{13}{115}\right)^3}{3} = 0.1 \cdot 10^{-2} > 0.25 \cdot 10^{-3}$$

$$n = 2 \Rightarrow R_5\left(\frac{13}{115}\right) \leq \frac{9}{4} \cdot \frac{\left(\frac{13}{115}\right)^5}{5} = 0.83 \cdot 10^{-5} < 0.25 \cdot 10^{-3}$$

$$\Rightarrow \ln\frac{51}{64} = -2\left(\frac{13}{115} + \frac{1}{3}\left(\frac{13}{115}\right)^3\right) = -2(0.11304 + 0.00049) = -0.22704$$

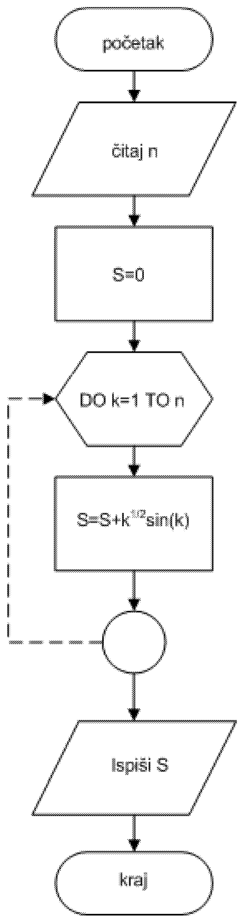
$$\Rightarrow \ln 51 = 6 \cdot 0.69315 - 0.22704 = 3.93186$$

$$\varepsilon = 0.83 \cdot 10^{-5} + 2 \cdot 0.5 \cdot 10^{-5} + 0 = 0.183 \cdot 10^{-4} < 10^{-3}.$$

Zadatak 2 *OOpišite dijagram toka i napišite program u Mathematica-i za algoritam koji za zadani cijeli broj* $n \geq 1$ *(ulazna informacija) računa*

$$\sin 1 + \sqrt{2} \cdot \sin 2 + \sqrt{3} \cdot \sin 3 \cdots + \sqrt{n} \cdot \sin n.$$

Rješenje.



Slika 1:

```
n = 100;
S = 0;
For[i = 1, i <= n, i = i + 1,
  S = S + N[√i * Sin[i]]];
Print[S]
```

-9.85149

Zadatak 3 *Jacobijevom metodom (jednom iteracijom) odredite približno rješenje sustava*

$$\begin{aligned} 3x_1 + 2x_2 &= -2 \\ x_1 + 4x_2 &= 5. \end{aligned}$$

Odredite pravu grešku.

Rješenje.

$$D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$\Rightarrow x^{(0)} = D^{-1}b = \frac{1}{12}\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{5}{4} \end{bmatrix}$$

$$\Rightarrow x^{(1)} = \frac{1}{12}\begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}\left(\begin{bmatrix} -2 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -\frac{2}{3} \\ \frac{5}{4} \end{bmatrix}\right) = \begin{bmatrix} -\frac{3}{2} \\ \frac{17}{12} \end{bmatrix}$$

Pravo rješenje:

$$\begin{aligned} 3x_1 + 2x_2 &= -2 \\ x_1 + 4x_2 &= 5 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 3x_1 + 2x_2 &= -2 \\ -3x_1 - 12x_2 &= -15 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{9}{5} \\ \frac{17}{10} \end{bmatrix}$$

Prava greška:

$$\varepsilon = \sqrt{\left(-\frac{3}{2} + \frac{9}{5}\right)^2 + \left(\frac{17}{12} - \frac{17}{10}\right)^2} = 0.41264$$

Zadatak 4 *Odredite vezu oblika* $y^2 = \frac{ax}{x+b}$ *ako je*

x_k	-0.9	-0.8	-0.7
y_k	4	3	2

Rješenje.

$$y^2 = \frac{ax}{x+b} \Rightarrow \frac{1}{y^2} = \frac{x+b}{ax} = \frac{1}{a} + \frac{b}{a} \cdot \frac{1}{x} \Rightarrow \bar{y} = a_0 + a_1\bar{x}, \quad \bar{y} = \frac{1}{y^2}, \bar{x} = \frac{1}{x}, a_0 = \frac{1}{a}, a_1 = \frac{b}{a}$$

$\bar{x}_i$	-1.11	-1.25	-1.43
$\bar{y}_i$	0.06	0.11	0.25

$$\Rightarrow \sum_{i=0}^{2} \bar{x}_i = -3.79, \sum_{i=0}^{2} \bar{x}_i^2 = 4.8395, \sum_{i=0}^{2} \bar{x}_i\bar{y}_i = -0.5616, \sum_{i=0}^{2} \bar{y}_i = 0.42 \Rightarrow a_0 = -0.62, a_1 = -0.6$$

$$\Rightarrow a = \frac{1}{a_0} = -1.61, \ b = a_1 a = 0.966 \Rightarrow y^2 = \frac{-1.61x}{x + 0.966}$$

Zadatak 5 *Odredite trigonometrijski polinom prvog stupnja koji u smislu metode najmanjih kvadrata najbolje aproksimira funkciju* $f(x) = \begin{cases} \frac{1}{2}, & x \in [0,1] \\ -\frac{1}{2}, & x \in (1,2] \end{cases}$. *Odredite kvadratnu grešku te aproksimacije.*

Rješenje.

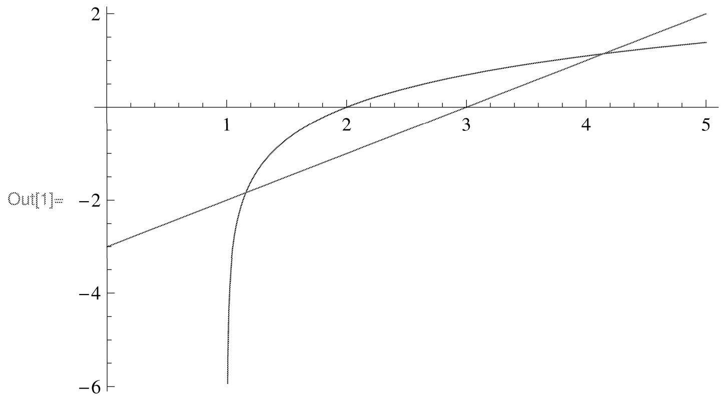
$$L = 1, \quad A_0 = \frac{1}{2}\int_0^1 \frac{1}{2}dx - \frac{1}{2}\int_1^2 \frac{1}{2}dx = 0$$

$$A_1 = \int_0^1 \frac{1}{2}\cos \pi x dx - \int_1^2 \frac{1}{2}\cos \pi x dx = 0, \ B_1 = \int_0^1 \frac{1}{2}\sin \pi x dx - \int_1^2 \frac{1}{2}\sin \pi x dx = \frac{2}{\pi}$$

$$T_1(x) = \frac{2}{\pi}\sin \pi x, \quad \varepsilon_1 = \int_0^2 \frac{1}{4}dx - 2 \cdot \frac{1}{2} \cdot \frac{4}{\pi^2} = 0.09471$$

Zadatak 6 *Pripremite za Newtonovu metodu i izračunajte prvu aproksimaciju manje nultočke jednadžbe* $x = \ln(x-1) + 3$.

Rješenje.



Slika 3:

$$f(x) = \ln(x-1) + 3 - x, \quad f(1.1) = -0.4 < 0, \quad f(2) = 1 > 0 \Rightarrow \text{ nultočka je unutar intervala } [1.1, 2]$$

$$f'(x) = \frac{1}{x-1} - 1 > 0, \quad f''(x) = -\frac{1}{(x-1)^2} < 0 \Rightarrow x_0 = 1.1$$

$$m_1 = \min \left| \frac{1}{x-1} - 1 \right| = 0.11, \quad M_2 = \max \left| \frac{1}{(x-1)^2} \right| = 100$$

$$|x_n - x_{n-1}| < \sqrt{\frac{2 \cdot 9 \cdot \varepsilon}{100}} = 0.18\varepsilon$$

$$x_1 = 1.1 - \frac{-0.4026}{9} = 1.14473$$

Zadatak 7 *Metodom iteracije s točnošću većom od* 0.1 *odredite približno rješenje sustava* $xy - y - 1 = 0,\ x^2 - y^2 - 1 = 0$ *uzimajući za početne vrijednosti* $x_0 = 2,\ y_0 = 1$.

Rješenje.

$$x = \frac{y+1}{y}, \quad y = \sqrt{x^2 - 1}$$

$$\begin{array}{ll} x_1 = 2, & y_1 = 1.732, \\ x_2 = 1.577, & y_2 = 1.732, \\ x_3 = 1.577, & y_3 = 1.219, \\ x_4 = 1.82, & y_4 = 1.219, \\ x_5 = 1.82, & y_5 = 1.521, \\ x_6 = 1.657, & y_6 = 1.521, \\ x_7 = 1.657, & y_7 = 1.321. \end{array}$$