Zadatak 1 *S točnošću većom od* 10^{-2} *odredite* $\ln 40$.

Rješenje.

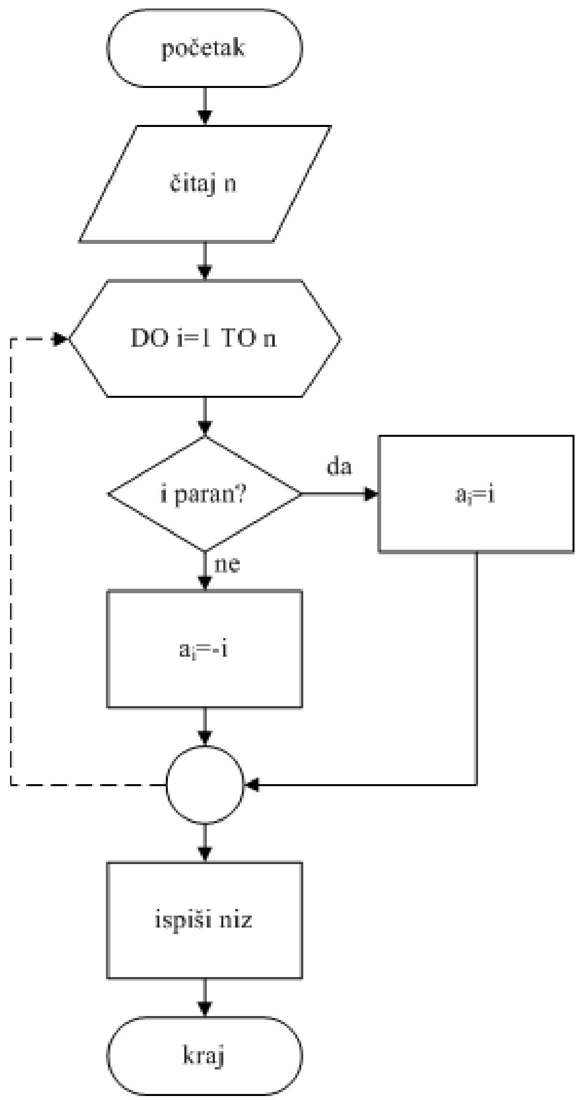
$$40 = 64 \cdot \frac{40}{64} = 2^6 \cdot \frac{5}{8}, \quad \ln 40 = 6\ln 2 + \ln\frac{5}{8}, \quad \frac{5}{8} = \frac{1-x}{1+x} \Rightarrow x = \frac{3}{13}$$

$$n = 2 \Rightarrow R_5\left(\frac{3}{13}\right) \leq \frac{9}{4}\frac{\left(\frac{3}{13}\right)^5}{5} = 0.2945 \cdot 10^{-3} < 0.25 \cdot 10^{-2}$$

$$\Rightarrow \ln\frac{5}{8} = -2\left(\frac{3}{13} + \left(\frac{3}{13}\right)^3 \cdot \frac{1}{3}\right) = -0.4698 \Rightarrow \ln 40 = 6 \cdot 0.6931 - 0.4698 = 3.6888$$

Zadatak 2 *Opišite dijagram toka i napišite program u Mathematica-i za algoritam koji formira niz duljine* n *t.d. je* $a_i = \begin{cases} i, & i \text{ paran} \\ -i, & i \text{ neparan} \end{cases}$.

Rješenje.




```
n = 5;
A = Table[0, {i, n}];
For[i = 1, i ≤ n, i = i + 1,
   If[IntegerQ[i/2] == True,
      A[[i]] = i,
      A[[i]] = -i
     ];
  ];
Print[A]

{-1, 2, -3, 4, -5}
```

Zadatak 3 *Odredite vezu oblika* $\frac{a}{x} + by^2 = 1$ *ako je*

x_k	0.08	0.20	1.00
y_k	4.95	3.05	1.10

Rješenje.

$$y^2 = \frac{1}{b} - \frac{a}{b}\frac{1}{x} \Rightarrow \bar{y} = a_0 + a_1\bar{x}, \quad \bar{y} = y^2, \bar{x} = \frac{1}{x}, a_0 = \frac{1}{b}, a_1 = -\frac{a}{b}$$

$\bar{x}_i$	1	5	12.5
$\bar{y}_i$	1.21	9.302	24.5

$$\Rightarrow \sum_{i=0}^{2} \bar{x}_i = 18.5, \sum_{i=0}^{2} \bar{x}_i^2 = 182.25, \sum_{i=0}^{2} \bar{x}_i\bar{y}_i = 353.97, \sum_{i=0}^{2} \bar{y}_i = 35.01 \Rightarrow a_0 = -0.821, a_1 = 2.025$$

$$\Rightarrow b = \frac{1}{a_0} = -1.218, \; a = -a_1 b = 2.47 \Rightarrow \frac{2.47}{x} - 1.218y^2 = 1$$

Zadatak 4 *Odredite trigonometrijski polinom prvog stupnja koji u smislu metode najmanjih kvadrata najbolje aproksimira funkciju* $f(x) = \begin{cases} 2, & x \in [0,1] \\ -2, & x \in (1,2] \end{cases}$. *Odredite kvadratnu grešku te aproksimacije*

Rješenje.

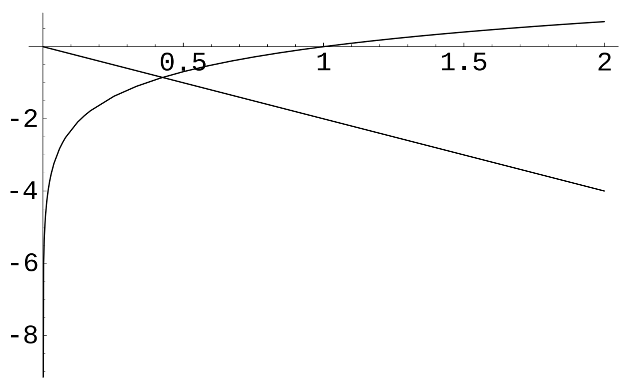
$$L = 1, \quad A_0 = \frac{1}{2}\int_0^1 2dx + \frac{1}{2}\int_1^2 (-2)dx = 0$$

$$A_1 = \int_0^1 2\cos\pi x dx + \int_1^2 (-2)\cos\pi x dx = 0, \; B_1 = \int_0^1 2\sin\pi x dx + \int_1^2 (-2)\sin\pi x dx = \frac{8}{\pi}$$

$$T_1(x) = \frac{8}{\pi}\sin\pi x, \quad \varepsilon_1 = \int_0^2 4dx - 2\cdot\frac{1}{2}\frac{64}{\pi^2} = 1.508864$$

Zadatak 5 *Za jednadžbu* $\ln x + 2x = 0$ *odredite funkciju* φ *s kojom se može provesti metoda iteracije.*

Rješenje.



Slika 2:

$$f(x) = \ln x + 2x, \quad f(0.1) = -2.1 > 0, \quad f(1) = 2 > 0 \Rightarrow \text{ nultočka je unutar intervala } [0.1, 1]$$

$$f'(x) = \frac{1}{x} + 2 \Rightarrow M_1 = \frac{1}{0.1} + 2 = 12 \Rightarrow \lambda < \frac{2}{12} = \frac{1}{6} \Rightarrow \lambda = \frac{1}{7} \Rightarrow \varphi(x) = x - \frac{1}{7}(\ln x + 2x)$$

Zadatak 6 *Newtonovom metodom (jednom iteracijom) odredite približno rješenje sustava* $x + 13\ln x - y^2 = 0,\ 2x^2 - xy - 5x + 1 = 0$ *uzimajući za početne vrijednosti* $x_0 = 1,\ y_0 = 0.$

Rješenje.

$$F(x,y) = \begin{bmatrix} x + 13\ln x - y^2 \\ 2x^2 - xy - 5x + 1 \end{bmatrix} \Rightarrow J(x,y) = \begin{bmatrix} 1 + \frac{13}{x} & -2y \\ 4x - y - 5 & -x \end{bmatrix}$$

$$\det J(x,y) = -x - 13 + 8xy - 2y^2 - 10y, \quad J^{-1}(x,y) = \frac{1}{\det J(x,y)} \begin{bmatrix} -x & 2y \\ -4x + y + 5 & 1 + \frac{13}{x} \end{bmatrix}$$

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \frac{1}{\det J(x_k, y_k)} \begin{bmatrix} -x_k & 2y_k \\ -4x_k + y_k + 5 & 1 + \frac{13}{x_k} \end{bmatrix} \cdot \begin{bmatrix} x_k + 13\ln x_k - y_k^2 \\ 2x_k^2 - x_k y_k - 5x_k + 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \frac{1}{-14} \begin{bmatrix} -1 & 0 \\ 1 & 14 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0.92857 \\ -1.92857 \end{bmatrix}$$

Zadatak 7 *Koristeći opći Newtonov oblik interpolacijskog polinoma odredite interpolacijski polinom za funkciju* $f(x) = \sin x$ *i čvorove* $x_0 = 0.30,\ x_1 = 0.35,\ x_2 = 0.40.$ *Izračunajte* $\sin 0.34$*, te uniformnu, lokalnu i pravu grešku.*

Rješenje.

x_i	y_i	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
$x_0 = 0.30$	$y_0 = 0.29552$		
		$f[x_0, x_1] = 0.9476$	
$x_1 = 0.35$	$y_1 = 0.3429$		$f[x_0, x_1, x_2] = -0.172$
		$f[x_1, x_2] = 0.9304$	
$x_2 = 0.40$	$y_2 = 0.38942$		

$$L_2(x) = 0.29552 + 0.9476(x - 0.30) - 0.172(x - 0.30)(x - 0.35) = -0.172x^2 + 1.0594x - 0.00682$$

Kako je sada $L_2(0.34) = 0.333493$ i $\sin 0.34 = 0.333487$ za pravu grešku imamo
$|L_2(0.34) - \sin 0.34| = 0.6 \cdot 10^{-5}$.

$$f'(x) = \cos x \Rightarrow f''(x) = -\sin x \Rightarrow f'''(x) = -\cos x \Rightarrow M_3(x) = \max_{x\in[0.30,0.35]} |\cos x| = 0.955336$$

Uniformna greška: $\frac{0.05^3}{4\cdot 3} 0.955336 = 0.99 \cdot 10^{-5}$

Lokalna greška: $\frac{0.955336}{6} |(0.34 - 0.30)(0.34 - 0.35)(0.34 - 0.40)| = 0.38 \cdot 10^{-5}$.