Zadatak 1 *S točnošću većom od* 10^{-4} *odredite* $\cos 615°$. *Izračunajte ukupnu grešku.*

Rješenje.

$$\cos 615° = \cos(360°+225°) = \cos 225° = \cos(180°+75°) = -\cos 75° = -\cos(90°-15°) = -\sin 15° = -\sin\frac{\pi}{12}$$

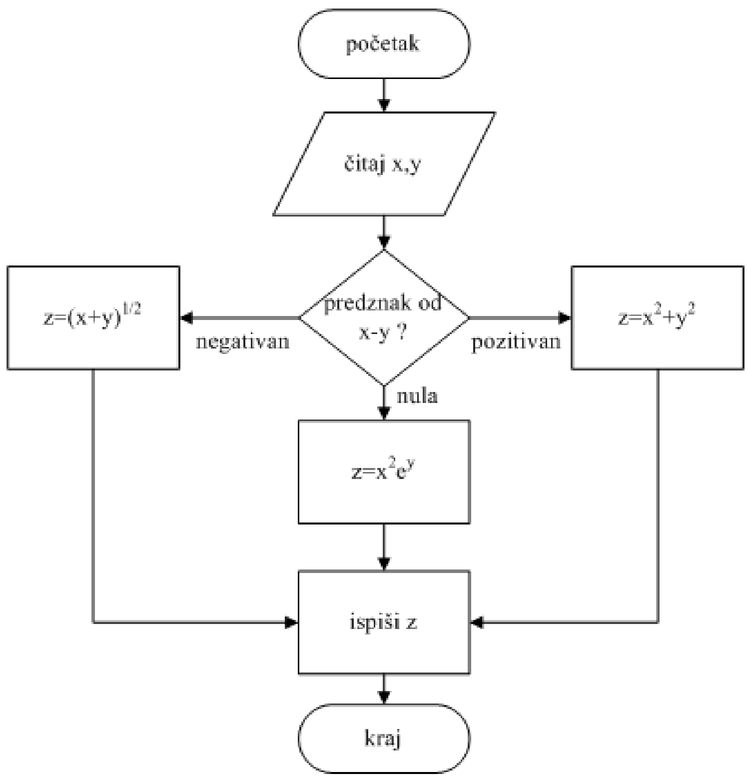
$$n = 2 \Rightarrow R_5\left(\frac{\pi}{12}\right) \le \frac{\left(\frac{\pi}{12}\right)^5}{5!} = 0.1 \cdot 10^{-4} < 0.25 \cdot 10^{-4}$$

$$\Rightarrow -\sin\frac{\pi}{12} = \frac{\pi}{12} - \frac{\left(\frac{\pi}{12}\right)^3}{3!} = 0.261799 - 0.002991 = 0.258808$$

$$\varepsilon = 0.1 \cdot 10^{-4} + 2 \cdot 0.5 \cdot 10^{-6} + 0 = 0.1001 \cdot 10^{-4} < 10^{-4}.$$

Zadatak 2 *Opišite dijagram toka i napišite program u Mathematica-i za algoritam koji za zadane x i y (ulazna informacija) računa vrijednost varijable z t.d. je* $z = \begin{cases} x^2 + y^2, & x > y \\ x^2 e^y, & x = y \\ \sqrt{x+y}, & x < y \end{cases}$.

Rješenje.



Slika 1:

```
In[1]:= x = 4;
        y = 5;
        If[x > y, z = x^2 + y^2,
          If[x == y, z = x^2 E^y, z = Sqrt[x + y]];
        ];
        Print[z]
```

Slika 2:

Zadatak 3 *Gauss-Seidelovom metodom (jednom iteracijom) odredite približno rješenje sustava*

$$\begin{aligned} 6x_1 + x_2 &= 9 \\ x_1 + 4x_2 &= 6. \end{aligned}$$

Odredite pravu grešku.

Rješenje.

$$L = \begin{bmatrix} 6 & 0 \\ 1 & 4 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$$

$$\Rightarrow x^{(0)} = L^{-1}b = \frac{1}{24}\begin{bmatrix} 4 & 0 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1.125 \end{bmatrix}$$

$$\Rightarrow x^{(1)} = \frac{1}{24}\begin{bmatrix} 4 & 0 \\ -1 & 6 \end{bmatrix}\left(\begin{bmatrix} 9 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1.5 \\ 1.125 \end{bmatrix}\right) = \begin{bmatrix} 1.3125 \\ 1.1719 \end{bmatrix}$$

Pravo rješenje:

$$\begin{aligned} 6x_1 + x_2 &= 9 \\ x_1 + 4x_2 &= 6 \end{aligned} \Leftrightarrow \begin{aligned} 6x_1 + x_2 &= 9 \\ -6x_1 - 24x_2 &= -36 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.3043 \\ 1.1739 \end{bmatrix}$$

Prava greška:

$$\varepsilon = \sqrt{(1.3125 - 1.3043)^2 + (1.1719 - 1.1739)^2} = 0.008$$

Zadatak 4 *Odredite vezu oblika* $a^x y^b = 2$ *ako je*

x_k	1	2	3
y_k	1.4	1.0	0.8

Rješenje.

$$x\ln a + b\ln y = \ln 2 \Rightarrow \ln y = \frac{\ln 2}{b} - \frac{\ln a}{b}x \Rightarrow \bar{y} = a_0 + a_1\bar{x}, \quad \bar{y} = \ln y, \bar{x} = x, a_0 = \frac{\ln 2}{b}, a_1 = -\frac{\ln a}{b}$$

$\bar{x}_i$	1	2	3
$\bar{y}_i$	0.34	0	−0.22

$$\Rightarrow \sum_{i=0}^{2}\bar{x}_i = 6, \sum_{i=0}^{2}\bar{x}_i^2 = 14, \sum_{i=0}^{2}\bar{x}_i\bar{y}_i = -0.32, \sum_{i=0}^{2}\bar{y}_i = 0.12 \Rightarrow a_0 = 0.6, a_1 = -0.28$$

$$\Rightarrow b = \frac{\ln 2}{a_0} = 1.15, \; a = e^{-a_1 b} = 1.38 \Rightarrow 1.38^x y^{1.15} = 2$$

Zadatak 5 *Odredite trigonometrijski polinom prvog stupnja koji u smislu metode najmanjih kvadrata najbolje aproksimira funkciju* $f(x) = \left|\frac{x}{2}\right|, \; x \in [-\pi, \pi]$. *Odredite kvadratnu grešku te aproksimacije.*

Rješenje.

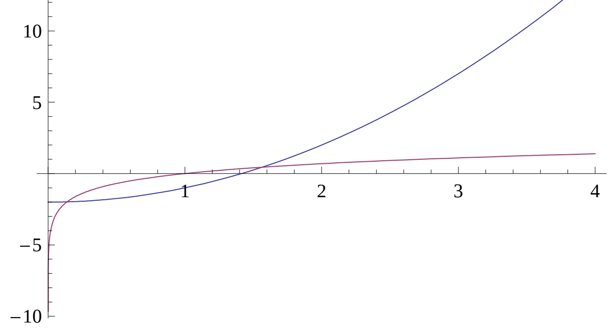
$$L = \pi, \quad A_0 = -\frac{1}{2\pi}\int_{-\pi}^{0}\frac{x}{2}dx + \frac{1}{2\pi}\int_{0}^{\pi}\frac{x}{2}dx = \frac{\pi}{4}$$

$$A_1 = -\frac{1}{\pi}\int_{-\pi}^{0}\frac{x}{2}\cos x dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{x}{2}\cos x dx = -\frac{2}{\pi}, \; B_1 = -\frac{1}{\pi}\int_{-\pi}^{0}\frac{x}{2}\sin x dx + \frac{1}{\pi}\int_{0}^{\pi}\frac{x}{2}\sin x dx = 0$$

$$T_1(x) = \frac{\pi}{4} - \frac{2}{\pi}\cos x, \quad \varepsilon_1 = \int_{-\pi}^{\pi}\frac{x^2}{4}dx - 2\pi\left[\frac{\pi^2}{16} + \frac{1}{2}\frac{4}{\pi^2}\right] = 0.0187$$

Zadatak 6 *Pripremite za Newtonovu metodu i izračnajte prvu aproksimaciju većeg rješenja za jednadžbu* $\ln x = x^2 - 2$.

Rješenje.



Slika 3:

$$f(x) = \ln x - x^2 + 2, \quad f(1) = 1 > 0, \quad f(2) = -1.31 < 0 \Rightarrow \text{ nultočka je unutar intervala } [1,2]$$

$$f'(x) = \frac{1}{x} - 2x < 0 \Rightarrow m_1 = |f'(1)| = 1 \Rightarrow f''(x) = -\frac{1}{x^2} - 2 < 0 \Rightarrow M_2 = |f''(1)| = 3$$

$$\Rightarrow \frac{3}{2}(x_n - x_{n-1})^2 < \varepsilon$$

$$x_0 = 2 \Rightarrow x_1 = 2 - \frac{f(2)}{f'(2)} = 1.62661$$

Zadatak 7 *Metodom iteracije s točnošću većom od* 10^{-3} *odredite približno rješenje sustava*

$$\begin{array}{rcl} x^3 + y^3 - 6x + 3 & = & 0 \\ x^3 - y^3 - 6y + 2 & = & 0 \end{array},$$

uzimajući za početne vrijednosti $x_0 = \frac{1}{2}, \ y_0 = \frac{1}{3}$.

Rješenje.

$$x = \frac{x^3 + y^3 + 3}{6} = \phi_1(x,y), \ y = \frac{x^3 - y^3 + 2}{6} = \phi_2(x,y)$$

$$x_1 = \phi_1(x_0, y_0) = 0.5270, \ y_1 = \phi_2(x_0, y_0) = 0.3480,$$

$$x_2 = \phi_1(x_1, y_1) = 0.5314, \ y_2 = \phi_2(x_1, y_1) = 0.3507,$$

$$x_3 = \phi_1(x_2, y_2) = 0.5322, \ y_3 = \phi_2(x_2, y_2) = 0.3511,$$

$$x_4 = \phi_1(x_3, y_3) = 0.5323, \ y_4 = \phi_2(x_3, y_3) = 0.3512.$$