Zadatak 1 *S točnošću većom od* 10^{-4} *odredite* $\ln 100$*. Izračunajte ukupnu grešku.*

Rješenje.

$$100 = 2^7 \cdot \frac{25}{32} \Rightarrow \ln 100 = 7\ln 2 + \ln \frac{25}{32}, \; \frac{25}{32} = \frac{1-x}{1+x} \Rightarrow x = \frac{7}{57}$$

$$n = 2 \Rightarrow R_5\left(\frac{7}{57}\right) \leq \frac{9}{4} \cdot \frac{\left(\frac{7}{57}\right)^5}{5} = 0.12569 \cdot 10^{-4} < 0.25 \cdot 10^{-4}$$

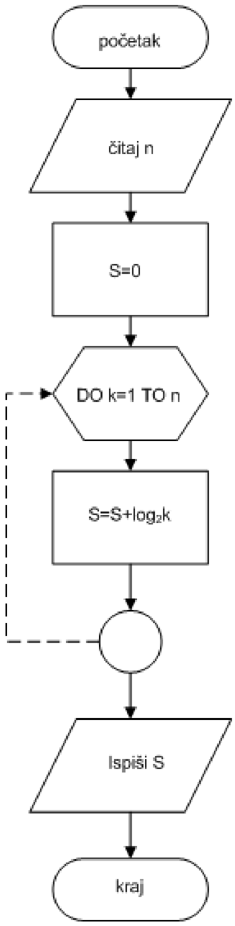
$$\Rightarrow \ln \frac{25}{32} = -2\left(\frac{7}{57} + \frac{1}{3}\left(\frac{7}{57}\right)^3\right) = -2(0.12281 + 0.00062) = -0.24686$$

$$\Rightarrow \ln 100 = 7 \cdot 0.69315 - 0.24686 = 4.60519$$

$$\varepsilon = 0.12569 \cdot 10^{-4} + 2 \cdot 0.5 \cdot 10^{-5} + 0 = 0.22569 \cdot 10^{-4} < 10^{-4}.$$

Zadatak 2 *Opišite dijagram toka i napišite program u Mathematica-i za algoritam koji za zadani cijeli broj* $n \geq 1$ *(ulazna informacija) računa* $\log_2 1 + \log_2 2 + \cdots + \log_2 n$*.*

Rješenje.



Slika 1:

```
n = 100;
S = 0;
For[k = 1, k ≤ n, k = k + 1,
  S = S + N[Log[2, k]]
 ];
Print[S]

524.765
```

Slika 2:

Zadatak 3 *Jacobijevom metodom (jednom iteracijom) odredite približno rješenje sustava*

$$\begin{aligned} 4x_1 + 2x_2 &= 5 \\ 2x_1 + 3x_2 &= 4. \end{aligned}$$

Odredite pravu grešku.

Rješenje.

$$D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

$$\Rightarrow x^{(0)} = D^{-1}b = \frac{1}{12}\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ \frac{4}{3} \end{bmatrix}$$

$$\Rightarrow x^{(1)} = \frac{1}{12}\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}\left(\begin{bmatrix} 5 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{5}{4} \\ \frac{4}{3} \end{bmatrix}\right) = \begin{bmatrix} \frac{7}{12} \\ \frac{1}{2} \end{bmatrix}$$

Pravo rješenje:

$$\begin{aligned} 4x_1 + 2x_2 &= 5 \\ 2x_1 + 3x_2 &= 4 \end{aligned} \Leftrightarrow \begin{aligned} 4x_1 + 2x_2 &= 5 \\ -4x_1 - 6x_2 &= -8 \end{aligned} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{7}{8} \\ \frac{3}{4} \end{bmatrix}$$

Prava greška:

$$\varepsilon = \sqrt{\left(\frac{7}{8} - \frac{7}{12}\right)^2 + \left(\frac{3}{4} - \frac{1}{2}\right)^2} = 0.38415$$

Zadatak 4 *Odredite vezu oblika* $a \ln x + b \ln y = 1$ *ako je*

x_k	1	3	8
y_k	0.35	10	180

Rješenje.

$$a \ln x + b \ln y = 1 \Rightarrow \ln y = \frac{1}{b} - \frac{a}{b} \ln x \Rightarrow \bar{y} = a_0 + a_1 \bar{x}, \quad \bar{y} = \ln y, \bar{x} = \ln x, a_0 = \frac{1}{b}, a_1 = -\frac{a}{b}$$

$\bar{x}_i$	0	1.098	2.079
$\bar{y}_i$	-1.05	2.302	5.193

$$\Rightarrow \sum_{i=0}^{2} \bar{x}_i = 3.177, \sum_{i=0}^{2} \bar{x}_i^2 = 5.528, \sum_{i=0}^{2} \bar{x}_i \bar{y}_i = 13.324, \sum_{i=0}^{2} \bar{y}_i = 6.445 \Rightarrow a_0 = -1.032, a_1 = 3.003$$

$$\Rightarrow b = \frac{1}{a_0} = -0.969, \ a = -a_1 b = 2.91 \Rightarrow 2.91 \ln x - 0.969 \ln y = 1$$

Zadatak 5 *Odredite trigonometrijski polinom prvog stupnja koji u smislu metode najmanjih kvadrata najbolje aproksimira funkciju* $f(x) = |x-2|, \ x \in [0,4]$. *Odredite kvadratnu grešku te aproksimacije.*

Rješenje.

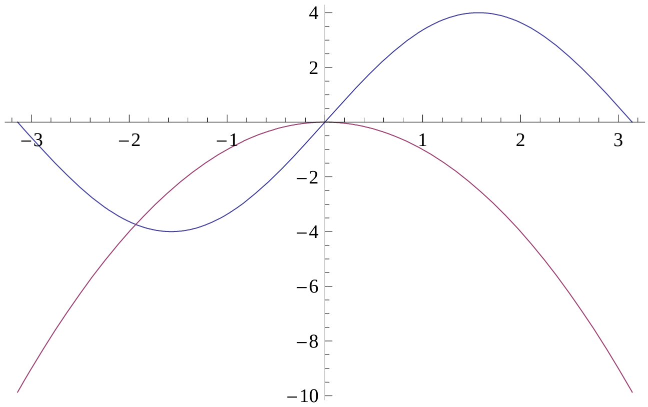
$$L = 2, \quad A_0 = \frac{1}{4}\int_0^2 (2-x)dx + \frac{1}{4}\int_2^4 (x-2)dx = 1$$

$$A_1 = \frac{1}{2}\int_0^2 (2-x)\cos\frac{\pi x}{2}dx + \frac{1}{2}\int_2^4 (x-2)\cos\frac{\pi x}{2}dx = \frac{8}{\pi^2}, \ B_1 = \frac{1}{2}\int_0^2 (2-x)\sin\frac{\pi x}{2}dx + \frac{1}{2}\int_0^4 (x-2)\sin\frac{\pi x}{2}dx = 0$$

$$T_1(x) = 1 + \frac{8}{\pi^2}\cos x, \quad \varepsilon_1 = \int_0^4 (x-2)^2 dx - 4\left[1 + \frac{1}{2}\frac{64}{\pi^4}\right] = 0.0193$$

Zadatak 6 *Pripremite za metodu sekante (jedan rub segmenta fiksiran) i izračnajte prvu aproksimaciju netrivijalnog rješenja za jednadžbu* $x^2 + 4\sin x = 0$.

Rješenje.



Slika 3:

$$f(x)=x^2{+}4\sin x,\;\; f(-\pi)=\pi^2>0,\;\; f\left(-\frac{\pi}{2}\right)=-1.53<0 \Rightarrow \text{ nultočka je unutar intervala } \left[-\pi,-\frac{\pi}{2}\right]$$

$$f'(x)=2x+4\cos x \Rightarrow m=\left|f'\left(-\frac{\pi}{2}\right)\right|=3.142,\;\; M=|f'(-\pi)|=10.283 \Rightarrow f''(x)=2-4\sin x>0$$

$$\Rightarrow \frac{7.141}{3.142}|x_n-x_{n-1}|<\varepsilon$$

$$x_0=-\frac{\pi}{2}\Rightarrow x_1=-\frac{\pi}{2}-\frac{\left(-\frac{\pi}{2}\right)^2-4}{\left(-\frac{\pi}{2}\right)^2-4-\pi^2}\left(-\frac{\pi}{2}+\pi\right)=-1.78193$$

Zadatak 7 *Metodom iteracije s točnošću većom od* 10^{-3} *odredite približno rješenje sustava*

$$\begin{array}{lcl} x^3y^2+17y & = & 1 \\ 9x-x^4\sin y^2 & = & 1 \end{array},$$

uzimajući za početne vrijednosti $x_0=y_0=0.5$.

Rješenje.

$$x=\frac{1+x^4\sin y^2}{9}=\phi_1(x,y),\;\; y=\frac{1-x^3y^2}{17}=\phi_2(x,y)$$

$$x_1=\phi_1(x_0,y_0)=0.11114,\;\; y_1=\phi_2(x_0,y_0)=0.05698,$$

$$x_2=\phi_1(x_1,y_1)=0.11111,\;\; y_2=\phi_2(x_1,y_1)=0.05882,$$

$$x_3=\phi_1(x_2,y_2)=0.11111,\;\; y_3=\phi_2(x_2,y_2)=0.05882.$$